Differentiation Rules:

Sum / Difference Rule: $\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

Power Rule: $\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$

Constant Multiple Rule: $\frac{d}{dx}\left(c\,f(x)\right) = c \cdot \frac{d}{dx}f(x)$

⊛ $\frac{d}{dx}(x) = 1$, $\frac{d}{dx}(\text{any constant}) = 0$

Eg. $h(x) = 5\sqrt{x^3} - 2\sqrt[3]{x^{11}}$. Find $h'(x)$.

$x^{1/n} = \sqrt[n]{x}$

$y^{2/7} = (y^2)^{1/7} = \sqrt[7]{y^2}$

$$h(x) = 5\,x^{3/2} - 2\,x^{\frac{11}{3}}$$

So $h'(x) = 5 \cdot \frac{3}{2}\,x^{\frac{3}{2}-1} - 2 \cdot \frac{11}{3}\,x^{\frac{11}{3}-1}$

$$= \frac{15}{2}\,x^{1/2} - \frac{22}{3}\,x^{\frac{8}{3}}$$

Eg. $g(x) = \frac{(x^3+4)(x^2+1)}{x}$

$$= \frac{x^3 \cdot x^2 + x^3 \cdot 1 + 4 \cdot x^2 + 4 \cdot 1}{x}$$

$$= \frac{x^5 + x^3 + 4x^2 + 4}{x}$$

$$= x^4 + x^2 + 4x + \frac{4}{x}$$

Then $g'(x) = \frac{d}{dx}\left(x^4 + x^2 + 4x + \frac{4}{x}\right)$

$$= 4x^3 + 2x + 4 \cdot 1 + 4\,\frac{d}{dx}\left(x^{-1}\right)$$

$$= 4x^3 + 2x + 4 + 4(-1)\,x^{-1-1}$$

$$= 4x^3 + 2x + 4 - \frac{4}{x^2}$$

Product Rule:

Let $A(x) =$ product of $f(x)$ & $g(x)$

$$= f(x) \cdot g(x)$$

Then, $A(x+h) = f(x+h) \cdot g(x+h)$

So, $A(x+h) - A(x)$

$$= f(x+h) \cdot g(x+h) - f(x) \cdot g(x)$$

$$= f(x+h) g(x+h) - f(x) \cdot g(x+h) + f(x) \cdot g(x+h) - f(x) \cdot g(x)$$

$$= [f(x+h) - f(x)] g(x+h) + f(x) [g(x+h) - g(x)]$$

So, $$\frac{A(x+h) - A(x)}{h} = \frac{[f(x+h) - f(x)] g(x+h) + f(x) [g(x+h) - g(x)]}{h}$$

$$= \frac{[f(x+h) - f(x)] g(x+h)}{h} + \frac{f(x) [g(x+h) - g(x)]}{h}$$

Then, $$\frac{d}{dx}(A(x)) = \lim_{h \to 0} \frac{A(x+h) - A(x)}{h}$$

$$\frac{d}{dx}[f(x) g(x)] = \lim_{h \to 0} \left\{ \frac{[f(x+h) - f(x)] g(x+h)}{h} + \frac{f(x) [g(x+h) - g(x)]}{h} \right\}$$

$$= \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \cdot g(x+h) \right] + \lim_{h \to 0} \left[f(x) \cdot \frac{g(x+h) - g(x)}{h} \right]$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \cdot \lim_{h \to 0} g(x+h) + f(x) \cdot \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{\overset{=hi}{f(x)}}{\underset{=lo}{g(x)}}\right] = \frac{g(x)\,f'(x) - g'(x)\cdot f(x)}{[g(x)]^2}$$

⊛ For Remembering: $\dfrac{\text{lo d hi} - \text{hi d lo}}{\text{lo lo}}$